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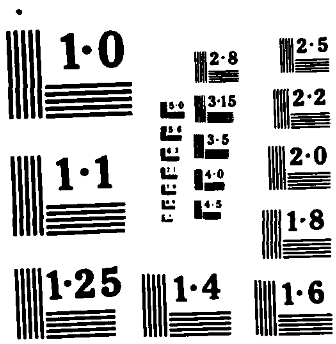

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In a recent paper it was shown that there are smooth, nonlinear, three-dimensional controllers, not incorporating probing signals, which are capable of adaptively stabilizing any single-input, single-output, minimum phase, relative degree two or less linear system of any dimension. Controllers of this type are based on minimal dynamic compensator synthesis. While such controllers are simple in structure they do not have a model-following capability.

This paper develops a new algorithm based on observer theory, which can adaptively stabilize and achieve model-following as well. The controller, which is a smooth nonlinear dynamical system of dimension 4(n+1), can adaptively stabilize any physical process with scalar input u and scalar output y, provided the process can be modelled by a strictly proper,

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INTRODUCTION

In a recent paper [1] it was shown that there are smooth, nonlinear, three-dimensional controllers, not incorporating probing signals, which are capable of adaptively stabilizing any single-input, single-output, minimum phase, relative degree two or less linear system of any dimension. Controllers of this type are based on minimal dynamic compensator synthesis [2]. While such controllers are simple in structure they do not have a model-following capability.

In this paper we develop a new algorithm based on observer theory [~~2~~], which can adaptively stabilize and achieve model-following as well. The controller, which is a smooth nonlinear dynamical system of dimension 4(n+1), can adaptively stabilize any physical process with scalar input u and scalar output y, provided the process can be modelled by a strictly-proper, minimum phase, linear system of dimension not exceeding n and relative degree not exceeding two. The controller is based on concepts developed previously in [1] and [3].

1. CONTROL EQUATIONS

The controller to be examined consists of a two-dimensional reference system

$$\begin{aligned} \dot{y}_r + \lambda_1 y_r &= \rho \\ \dot{\rho} + \lambda_2 \rho &= r \end{aligned} \tag{1}$$

where λ_1 and λ_2 are positive constants and $r(\cdot)$ is a bounded, differentiable reference input, a tracking error

$$e = y - y_r, \tag{2}$$

sensitivity function n-vectors θ_u and θ_y generated by the equations

$$\begin{aligned} \dot{\theta}_u &= A\theta_u + bu \\ \dot{\theta}_y &= A\theta_y + by \end{aligned} \tag{3}$$

where n is a prespecified positive integer and (A,b) is any n-dimensional controllable pair with A stable, and a control law

$$u = N(\|k\|)(k_u'\theta_u + k_y'\theta_y + k_\rho \rho + k_r r) \tag{4}$$

where k_u, k_y, k and k_r are control parameters,

$$k = [k_u', k_y', k_\rho, k_r]', \tag{5}$$

$\|k\| = (k'k)^{\frac{1}{2}}$, and $N(\cdot)$ is a Nussbaum Gain, i.e. any integrable function satisfying

$$\sup_{\xi>0}\left\{\frac{1}{\xi}\int_0^{\xi}\mu N(\mu)d\mu\right\} = \infty$$
$$\inf_{\xi>0}\left\{\frac{1}{\xi}\int_0^{\xi}\mu N(\mu)d\mu\right\} = -\infty \tag{6}$$

{e.g., $N(\mu) = \mu^2 \cos(\mu^2)$. Control parameters k_u, k_y, k_ρ and k_r are adjusted according to the rules

$$\begin{aligned}
k_u &= \theta_u e + z_u & \dot{z}_u &= \lambda_1 \theta_u e - (A\theta_u + bu)e \\
k_y &= \theta_y e + z_y & \dot{z}_y &= \lambda_1 \theta_y e - (A\theta_y + by)e \\
k_\rho &= \rho e + z_\rho & \dot{z}_\rho &= \lambda_1 \rho e - (r - \lambda_2 \rho)e \\
k_r &= re + z_r & \dot{z}_r &= \lambda_1 re - \dot{r}e
\end{aligned} \tag{7}$$

The controller defined by (1)-(7) may be viewed as a smooth $4(n+1)$-dimensional dynamical system with inputs r, $\dot{r}$ and y, state $\{y_r, \rho, \theta_u, \theta_y, z_u, z_y, z_\rho, z_r\}$ and output u.

Remarks:

1. By using minimal dimensional observer theory, it is possible to reduce the dimensions of θ_u and θ_y to $(n-1)$ and to eliminate one control parameter thereby obtaining a $(4n+1)$-dimensional algorithm with the same capabilities as the one described here [2]. The stability analysis of the lower-dimensional algorithm is essentially the same as the analysis which follows.

2. There are two different ways to avoid generating the derivative of the reference signal r required by the above algorithm. The first is simply to introduce a new reference signal $\bar{r}$ and then make r a state of a three-dimensional reference system defined by (1) and $\dot{r} + \lambda_3 r = \bar{r}$ where $\lambda_3 > 0$. The second is to make use of an idea due to Monopoli [4,5] which amounts to replacing (4) and (7) by

$$u = N(\|k\|)k'\theta + \left(\frac{(k'\phi)^2}{2\|k\|}\frac{\partial N(\|k\|)}{\partial \|k\|} + N(\|k\|)\phi'\phi\right)e$$

and

$$\dot{k} = \phi e$$

respectively where

$$\theta = [\theta_u', \theta_y', \rho, r]' \tag{8}$$

and $\dot{\phi} + \lambda_2 \phi = \theta$. This alternative, however, requires $N(\cdot)$ to be differentiable.

2. MAIN RESULT

The process to which the preceding algorithm is applicable must admit a minimum phase, linear model of dimension $\tilde{n} \leq n$ and relative degree ≤ 2. It is known [6] that these assumptions imply that such a process can also be modelled by a n-dimensional, stabilizable, minimum phase, relative degree two or one linear system of the form

$$\begin{aligned}
\dot{x}_p &= (A + h_p c)x_p + b_p u \\
y &= cx_p
\end{aligned} \tag{9}$$

where c is any row vector chosen so that (c,A) is observable, and k_p and b_p are unknown constant, parameter vectors. We take (9) as the model of the process to be controlled. Our main result is as follows.

Theorem 1: For each initial state and each differentiable input r, bounded on $[0,\infty)$, the state response $X = \{x_p, \theta_u, \theta_y, \rho, y_r, z_u, z_y, z_\rho, z_r\}$ of the closed-loop system defined by (1)-(9) exists and is bounded on $[0,\infty)$ and the tracking error $e \to 0$ as $t \to \infty$.

The remainder of this paper is devoted to a proof of this theorem.

3. STABILITY ANALYSIS

To prove Theorem 1, it proves useful to work with a certain system of equations which we now derive. For this let n^* denote the relative degree of (9) and define

$$\delta = \dot{y} + \lambda_1 y \tag{10}$$

It is known [6] that for $n^* = 1$,

$$\delta = gu - d_u'\theta_u - d_y'\theta_y + \varepsilon \tag{11}$$

where g is a nonzero constant - the "high-frequency gain" - d_u and d_y are unknown parameter vectors, and ε is an unknown linear combination of decaying exponentials. Similarly, for $n^* = 2$ it is known that

$$\dot{\delta} + \lambda_2\delta = gu - d_u'\theta_u - d_y'\theta_y + \varepsilon \tag{12}$$

where g, d_u, d_y and ε have the same interpretations as for $n^* = 1$. It is also known [6] in either case that if $\bar{x} = [\theta_u', \theta_y', x_p']'$, then $\bar{x}$ satisfies

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{b}(gu - d_u'\theta_u - d_y'\theta_y + \varepsilon) \tag{13}$$

where $\bar{A}$ is stable.

To proceed, define

$$\sigma = \dot{e} + \lambda_1 e \tag{14}$$

and note from (1), (2) and (10) that $\sigma = \delta - \rho$. It follows from (1), (11) and (12) that

$$\sigma = gu - d_u'\theta_u - d_y'\theta_y + \varepsilon - \rho$$

if $n^* = 1$ and

$$\dot{\sigma} + \lambda_2\sigma = gu - d_u'\theta_u - d_y'\theta_y + \varepsilon - r$$

if $n^* = 2$. Thus if we define $d = [d_u', d_y', 1, 0]'$ for $n^* = 1$ or $d = [d_u', d_y', 0, 1]'$ for $n^* = 2$, then

$$\sigma = gu - d'\theta + \varepsilon \quad \text{if } n^* = 1 \tag{15}$$

and

$$\dot{\sigma} + \lambda_2\sigma = gu - d'\theta + \varepsilon \quad \text{if } n^* = 2 \tag{16}$$

where θ was defined previously in (8). If in addition we define $q = [1,0]'$ for $n^* = 1$ or $q = [0,1]'$ for $n^* = 2$, then for $n^* = 1$ or 2 (13) becomes

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{b}(gu - d'\theta + \varepsilon + [\rho, r]q) \tag{17}$$

Thus with the notation

$$\bar{u} = gu - d'\theta + \varepsilon,$$

(15) through (17) can be rewritten as

$$\sigma = \bar{u} \qquad \text{if } n^* = 1 \tag{15$'$}$$

$$\dot{\sigma} + \lambda_2\sigma = \bar{u} \qquad \text{if } n^* = 2 \tag{16$'$}$$

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{b}(\bar{u} + [\rho,r]'q) \tag{17$'$}$$

By differentiating the expressions for k_u, k_y, k_ρ and k_r in (7) and substituting in the expressions for $\dot{z}_u, \dot{z}_y, \dot{z}_\rho, \dot{z}_r$ also in (7), it is straightforward to verify that $\dot{k}_u = \theta_u\sigma$, $\dot{k}_y = \theta_y\sigma$, $\dot{k}_\rho = \rho\sigma$ and $\dot{k}_r = r\sigma$, σ being given by (14). Using (5) and (8) we can thus write

$$\dot{k} = \theta\sigma \tag{18}$$

For ease of reference we now summarize in one place, the system of equations to be analyzed.

$$\dot{e} + \lambda_1 e = \sigma \tag{19a}$$

$$\sigma = \bar{u} \qquad \text{if } n^* = 1 \tag{19b}$$

$$\dot{\sigma} + \lambda_2\sigma = \bar{u} \qquad \text{if } n^* = 2 \tag{19c}$$

$$\bar{u} = gN(\|k\|)k'\theta - d'\theta + \varepsilon \tag{19d}$$

$$\dot{k} = \theta\sigma \tag{19e}$$

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{b}(\bar{u} + [\rho,r]'q) \tag{19f}$$

$$\dot{\rho} + \lambda_2\rho = r \tag{19g}$$

$$\theta = [\theta_u', \theta_y', \rho, r]' \tag{19h}$$

$$\bar{x} = [\theta_u', \theta_y', x_p']' \tag{19i}$$

The preceeding defines a dynamical system of the form $\dot{Z} = F(Z,r,\varepsilon)$ where $Z = [e,\sigma,k,\bar{x},\rho]'$ (with σ deleted from Z if $n^* = 1$). Observe that boundedness of Z implies boundedness of $e,\sigma,k,\theta_u,\theta_y,x_p$ and ρ. Boundedness of ρ together with (1) implies boundedness of y_r. In addition, boundedness of $e,k,\theta_y,\theta_y,\rho$ and r together with (5) and (7) imply boundedness of z_u,z_y,z_ρ and z_r. Thus to prove Theorem 1 it is enough to show that Z(t) exists and is bounded on $[0,\infty)$ and that $e(t) \to 0$ as $t \to \infty$.

Differentiability of F implies that for any initial state Z(0), there must exist an interval $I = [0,t_1)$ on which a solution Z(t) exists. For any function $\xi(t)$ defined on I, we write $\xi \in L^\infty(I)$ if $\|\xi\| = \sqrt{\xi'\xi}$ is bounded on I by a constant not depending on t_1; we also write $\xi \in L^2(I)$ if $\int_0^t \|\xi(\tau)\|^2 d\tau \in L^\infty(I)$.

We can now state

<u>Proposition</u> 1: <u>There exist constants</u> C_1 <u>and</u> C_2, <u>not depending on</u> t_1, <u>such that for</u> $t \in I$

$$\int_0^t \sigma(\tau)\bar{u}(\tau)d\tau \leq g\pi(\|k(t)\|) + \frac{\lambda_2}{1+\lambda_2}\int_0^t \sigma^2(\tau)d\tau + C_1\|k(t)\| + C_2 \tag{20}$$

<u>where</u>

$$\pi(\xi) = \int_0^\xi \mu N(\mu)d\mu \tag{21}$$

<u>Proof</u>: Set $\eta = \|k\|^2$; hence from (19e), $\dot{\eta} = 2k'\theta\sigma$. It follows from this, (19d) and (19e) that

$$\sigma\bar{u} = \frac{g}{2}N(\eta^{\frac{1}{2}})\dot{\eta} - d'\dot{k} + \sigma\varepsilon \tag{22}$$

We can now develop bounds for the integrals of the terms on the right side of (22).

First note that

$$\begin{aligned} g/2\int_0^t N(\eta^{\frac{1}{2}}(\tau))\dot{\eta}(\tau)d\tau &= g/2\int_{\eta(0)}^{\eta(t)} N(\eta^{\frac{1}{2}})d\eta \\ &= g\int_{\|k(0)\|}^{\|k(t)\|}\omega N(\omega)d\omega \\ &= g\pi(\|k(t)\|) - g\pi(\|k(0)\|) \end{aligned} \tag{23}$$

Next observe that

$$\begin{aligned} -\int_0^t d'\dot{k}(\tau)d\tau &= d'k(0) - d'k(t) \\ &\leq \|d\|\,\|k(t)\| + |d'k(0)| \end{aligned} \tag{24}$$

For any positive constant C, $\sigma\,\varepsilon \leq C\sigma^2 + \varepsilon^2/4C$. Thus

$$\int_0^t \sigma(\tau)\varepsilon(\tau)d\tau \leq C\int_0^t \sigma^2(\tau)d\tau + \tfrac{1}{4}C\int_0^t \varepsilon^2(\tau)d\tau \tag{25}$$

By setting $C = \lambda_2/(1+\lambda_2)$, $C_1 = \|d\|$ and

$$C_2 = |g\pi(\|k(0)\|)| + |d'k(0)| + \tfrac{1}{4}C\int_0^\infty \varepsilon^2(\tau)d\tau,$$

(23)-(25) can be combined to yield (20). ∇

<u>Proof of Theorem</u> 1: From (19b) and (19c) it follows that $\int_0^t \sigma(\tau)\bar{u}(\tau)d\tau$ equals $\int_0^t \sigma^2(\tau)d\tau$ if $n^* = 1$ or $\lambda_2\int_0^t \sigma^2(\tau)d\tau + \frac{1}{2}(\sigma^2(t) - \sigma^2(0))$ if $n^* = 2$. This and Proposition 1 thus imply that

$$\Omega(\|k(t)\|) \geq \left\{\begin{array}{lll} \dfrac{1}{(1+\lambda_1)} \displaystyle\int_0^t \sigma^2(\tau)d\tau & & \text{if } n^* = 1 \\ \dfrac{\lambda_1^2}{(1+\lambda_1)} \displaystyle\int_0^t \sigma^2(\tau)d\tau & + \dfrac{\sigma^2(t)}{2} & \text{if } n^* = 2 \end{array}\right\} \tag{26}$$

where

$$\Omega(\xi) = g\pi(\xi) + C_1\xi + C_2 + \sigma^2(0)/2$$

In view of (6) and the definition of $\pi(\xi)$ in (21) it is easy to see that there must exist a closed-bounded interval $[a,b]$ containing $\|k(0)\|$ for which both $\Omega(a)$ and

$\Omega(b)$ are negative. Since (26) implies that for any $t_\infty \varepsilon I$, $\Omega(\|k(t)\|) \geq 0$, $\|k(t)\|$ cannot pass through either a or b. Therefore $\|k\| \varepsilon_2 L^\infty(I)$. In addition, since $\Omega(\xi)$ is continuous, it follows from (26) that $\sigma \varepsilon L^2(I)$ for $n^* = 1$ and $\sigma \varepsilon L^2(I) \cap L^\infty(I)$ for $n^* = 2$. Thus $\sigma \varepsilon L^2(I)$ for $n^* = 1,2$ so by (19a) $e \varepsilon L^2(I) \cap L^\infty(I)$ for $n^* = 1,2$.

For $n^* = 1$, $\bar{u} = \sigma \varepsilon L^2(I)$; since $[\rho,r]' \varepsilon L^\infty(I)$ it follows from (19f) that $\bar{x} \varepsilon L^\infty(I)$.

For $n^* = 2$, $\bar{u} = \dot{\sigma} + \lambda_2\sigma$ and $\sigma \varepsilon L^\infty(I)$; again it follows from (19f) that $\bar{x} \varepsilon L^\infty(I)$.

At this point we have shown that $Z = [e,\sigma,k,\bar{x},\rho]' \varepsilon L^\infty(I)$ for $n^* = 2$, that $Z = [e,k,\bar{x},\rho]' \varepsilon L^\infty(I)$ for $n^* = 1$ and that $e \varepsilon L^2[I]$ for $n^* = 1,2$. Therefore we can take $t_1 = \infty$. Thus Z is bounded on $[0,\infty)$ and $e \varepsilon L^2[0,\infty)$. In addition, since (19) implies that $\dot{e} \varepsilon L^\infty(0,\infty)$, it follows that $e \to 0$ as $t \to \infty$. ∇

CONCLUDING REMARKS

The algorithm presented here and its subsequent stability analysis rely for the most part on ideas developed previously in [1], [3] and [6]. In fact the stability analysis given is almost identical to that used in [1]. The essential new idea in this paper is to use a control law (4) incorporating <u>both</u> reference model state ρ and reference input r. It is this departure from more traditional adaptive control laws, (e.g. [3]) which makes model following possible with <u>one</u> algorithm for processes of <u>both</u> relative degree 1 and 2.

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FOOTNOTES

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